EVALUATING THE STRENGTH OF A WELD WITH A SMALL DISK-SHAPED CRACK

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It is known [1-3] that the heat-affected zone (HAZ) of welds, due to features of the welding operation, is subject to nonuniform, continuously varying stresses. Due to the action of these thermal stresses and structural-phase processing stresses, the metal of the HAZ undergoes plastic strains which lowers its strain-hardening capacity during laoding in service. Also, these factors may be the cause of processing-induced cracks in the weld.

The present study examines the problem of evaluating the effect of a small disk-shaped processing-induced crack on weld strength. The crack is located in the plane of a thin HAZ acted upon by external stresses. Here, the character of the behavior of the metal in the HAZ can be associated with the following properties, according to the data in [1-4]:

— the elastic characteristics of the HAZ metal are equal to the elastic characteristics of the surrounding volumes of metal, while the strength characteristics are lower. This means that plastic strain may be localized in the relatively thin layer of HAZ metal while the surrounding volumes of metal remain elastic;

— with uniaxial tension of the weld in the direction perpendicular to the plane of the HAZ (given the absence of initial cracks in the HAZ) under conditions of ideal stress-controlled loading, fracture of the HAZ metal occurs after a small amount of strain-hardening when the ultimate strength σ_u is reached in it; under conditions of ideal strain-controlled loading, attainment of the ultimate strength with a further separation of the boundaries of the HAZ is followed by a smooth decrease in the load connected with the accumulation of strains in the HAZ metal;

— with uniaxial tension of the weld in the direction perpendicular to the HAZ under conditions intermediate between ideal stress- and ideal strain-controlled loading, after the HAZ metal reaches its ultimate strength it behaves initially as in the case of ideal strain-controlled loading; it fractures after a certain strain level is reached, as in the case of ideal stress-controlled loading.

When examining the behavior of the HAZ metal at the edge of the crack, it is expedient to proceed on the basis of the complete curve-describing the force versus the displacement of the boundaries of the HAZ. This curve terminates in a smoothly decreasing (to zero) softening section. Such a curve is shown in Fig. 1a by curve 1. Here, we have used the following notation: h is the initial thickness of the HAZ; Δh is the separation of the boundaries of the HAZ; q are the forces applied along a normal to the boundaries of the HAZ, referred to a unit area of the cross section; σ_u is the ultimate strength of the HAZ metal; E is the elastic modulus of the HAZ metal. With a reduced strain-hardening capacity, the actual curve can be approximated by a piecewise-linear curve of the form 2 in Fig. 1a consisting of two sections. One section corresponds to nominally elastic behavior of the HAZ metal until attainment of the ultimate strength σ_u, while the other corresponds to linear softening of the HAZ metal after the passage of σ_u. A good approximation can be obtained in this case if it is based on equality of the areas under the actual and approximating curves.

By virtue of the above assumptions regarding the behavior of HAZ metal, the solution of the stated problem of evaluating the static strength of a weld with a small disk-shaped crack can be constructed on the basis of a model of linear softening metal in a thin plastic-strain zone adjacent to the edge of the crack [5-13]. To do this, we need to replot the initial curves relative to the irreversible displacements $\delta = \Delta h - h\sigma_u/E$ of the boundaries of the HAZ with respect to each other, as shown in Fig. 1b. Then the idealized curve will

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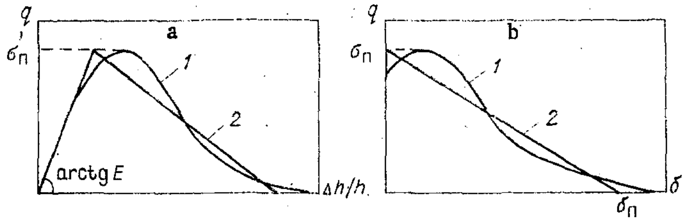

Fig. 1

retain only the descending softening section, shown in Fig. 1b by line 2. The equation of this line has the form

$$q = \sigma_u(1 - \delta/\delta_u),\ 0 \leqslant \delta \leqslant \delta_u. \tag{1}$$

The quantity δ_u introduced here has the meaning of the limiting irreversible separation of the HAZ boundaries at which a fracture surface appears under conditions of ideally-strain-controlled loading after a smooth decrease in the forces to zero. It is a constant of the HAZ metal for the given welding technology and should be determined from the condition of equality of the areas under the real and approximating stress—strain curves. At the same time, the area under the forces — irreversible displacement curve determines the work expended on fracture of a specimen of unit cross section cut out perpendicular to the HAZ, or the energy necessary to form two new fracture surfaces in the failed specimens (see, e.g., [14, 15]). Using δ to designate the surface energy necessary to form a unit of new surface and using Eq. (1) to calculate the fracture work, it is not hard to find that

$$\delta_u = 4\gamma/\sigma_u. \tag{2}$$

In the absence of the empirically hard-to-obtain record of the total curve showing force against displacements of the HAZ boundaries under conditions of ideal strain-controlled loading, derived equation (2) makes it possible to evaluate the constant σ_u in Eq. (1) through the surface energy γ. The method for experimental determination of the latter is well established. Also, in the presence of experimental values of the critical stress intensity factor K_{Ic} obtained on sufficiently large specimens with a crack oriented along the HAZ, it is possible to use (2) (see, e.g., [14, 15]) to evaluate δ_u through the quantity K_{Ic}:

$$\delta_u = 2(1-\nu^2)\frac{\sigma_u}{E}\left(\frac{K_{Ic}}{\sigma_u}\right)^2, \tag{3}$$

where ν is the Poisson's ratio. Thus, the quantity δ_u introduced in (1) is connected with the fracture toughness of the HAZ metal and turns out to be greater, the greater this toughness. Here, we will use Eqs. (1) and (3) to formulate the methods proposed in [11-13] to solve a problem involving determination of the stress state and stability of softening HAZ metal near the front of a disk-shaped processing-induced crack located in the plane of the HAZ.

We will represent the weld with HAZ in the form of an infinite circular medium with an elastic modulus E and a Poisson's ratio ν containing a planar crack having the form of a circle of radius a in plan. Let the medium be subjected at infinity to tension by a uniformly distributed external stress p in the direction perpendicular to the crack plane. We will select the cylindrical coordinate system $Ozr\varphi$ shown in Fig. 2. It is assumed that during loading a thin annular zone of weakened bonds (ZWB) appears at the edge of the disk-shaped crack. This zone is characterized by the presence of irreversible separations δ of the boundaries of the HAZ. The radial dimension d of this zone depends on the magnitude of the applied load, which is unknown beforehand and must be found in the course of solving the problem. The actual crack, of radius a, is nominally increased in size by an amount corresponding to the size of the ZWB so that the radius of the imaginary disk-shaped crack becomes equal to $b = a + d$. It is assumed that the edges of the imaginary crack on the ZWB section from a to b interact with each other in accordance with Eq. (1), where $q = q(r)$ and $\delta = \delta(r)$ represent the forces of interaction and the separation of the edges of the imaginary crack. We know the following expression for separation of the crack edges from the solution of the problems of a planar circular crack of radius b, the surface of which is subjected to normal self-balanced axisymmetric loads s(r) (see, e.g., [16]):

$$\delta(r) = \frac{8(1-\nu^2)}{\pi E}\int_0^{\pi/2}\int_{r\sin\alpha}^{b\sin\alpha}\frac{s(t)t\,dtd\alpha}{\sqrt{t^2 - r^2\sin^2\alpha}},\quad 0 \leqslant r \leqslant b. \tag{4}$$

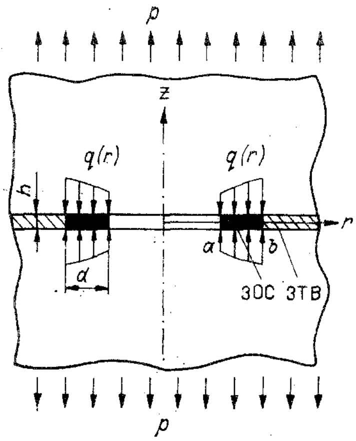

Fig. 2

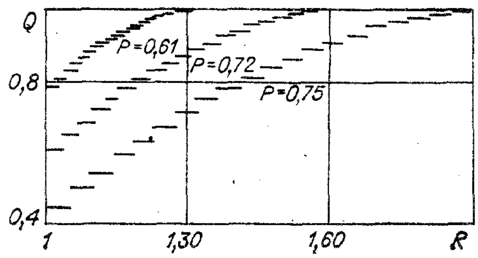

Fig. 3

For the problem being examined here, the load s(t) is determined from the formula

$$s(t) = \begin{cases} p & \text{for} \quad 0 \leqslant t \leqslant a, \\ p - q(t) & \text{for} \quad a \leqslant t \leqslant b. \end{cases} \tag{5}$$

Inserting (1) and (5) into (4), we obtain an integral equation to determine the forces of interaction q(r) of the edges of the imaginary crack on the ZWB section from a to b:

$$q(r) - \frac{1}{m}\left[\int\limits_{\arcsin\frac{a}{b}}^{\arcsin\frac{a}{r}}\int\limits_{a}^{b\sin\alpha}\frac{q(t)t\,dt\,d\alpha}{\sqrt{t^2-r^2\sin^2\alpha}} + \int\limits_{\arcsin\frac{a}{r}}^{\pi/2}\int\limits_{r\sin\alpha}^{b\sin\alpha}\frac{q(t)\,t\,dt\,d\alpha}{\sqrt{t^2-r^2\sin^2\alpha}}\right] = \sigma_u - \frac{1}{m}p\sqrt{b^2-r^2} \quad \text{at} \quad a \leqslant r \leqslant b, \tag{6}$$

where

$$m = \pi E\delta_u/[8(1-\nu^2)\sigma_u]. \tag{7}$$

The coefficient m in this equation is a constant of the HAZ metal for a given welding technology, has the dimension of length, and with allowance for Eq. (3) is expressed in the form

$$m = (\pi/4)(K_{Ic}/\sigma_u)^2, \tag{8}$$

i.e., it is a characterstic of the fracture toughness of the HAZ metal. To determine the unknown dimension b in Eq. (6), it must be agumented by the condition of smoothness of the joining (closure) of the edges of the imaginary crack:

$$d\delta(r)/dr|_{r=b} = 0.$$

With allowance for (4) and (5), this condition is written in the form

$$pb = \int\limits_a^b \frac{q(t)\,tdt}{\sqrt{b^2-t^2}}. \tag{9}$$

Derived integral equation (6) is a second-order Fredholm equation with a positively determined kernel. It has a unique solution with an arbitrary right side if the coefficient m does not coincide with any of the eigenvalues m_i ($m_i \geq m_{i+1} > 0$, $i = 1, 2, 3, \ldots$) corresponding to the homogeneous integral equation. Since the dimension b is present in the integral terms of Eq. (6), each eigenvalue of the homogeneous integral equation is actually a monotonically increasing function of the dimension b or of the associated dimension d = b − a. An increase in the applied load p, in accordance with (9), entails an increase in the dimension b (or d). The latter in turn leads to an increase in the eigenvalues m_i of the corresponding homogeneous integral equation. Here, as long as the first maximum eigenvalue m_1 is less than the material constant m, integral equation (6) will have a solution with an arbitrary right side. The necessary condition for the existence of a solution to integral equation (6) is first violated when m_1 becomes equal to m. According to [11-13], this moment is identical to the moment of loss of stability of the crack in the initial physical problem.*

*The corresponding critical values of the sought quantities are designated below by an asterisk.

TABLE 1

D	0,100	0,200	0,300	0,400	0,500	0,600	0,700	0,800	0,900	1,000
M_1	0,106	0,210	0,313	0,413	0,512	0,610	0,707	0,802	0,896	0,990

The eigenvalue m_1 of the homogeneous integral equation corresponding to integral equation (6) was determined and Eq. (6) was solved together with auxiliary conditions (9) with $m_1 \neq m$ by numerical relative to the dimensionless quantities:

$$R = r/a,\ T = t/a,\ B = b/a,\ D = d/a,\ Q = q/\sigma_{\text{п}},\ P = p/\sigma_{\text{п}},\quad M = m/a,\quad M_1 = m_1/a.$$

Here, the segment [1, B] was broken down into 20 intervals. It was assumed that in each interval the sought stress Q(R) is constant, and integral equation (6) was accordingly approximated for each specific value of B by a system of twentieth-order linear algebraic equations. The integrals over α entering into the determination of the coefficients of this system were calculated from the Gauss integration formula to within three signs, inclusively. We then calculated the eigenvalue M_1 corresponding to a homogeneous approximating system for each specified value by B by the Kellog method of successive approximations [17]. The calculations were performed to within three signs, inclusively. The approximating system was solved for specified values of $M \neq M_1$ by the Gauss elimination method. The solution was obtained separately for the first and second terms (with P = 1) of the right side of Eq. (6), and we subsequently formed a linear combination of the two solutions with an unknown multiplier P ahead of the second term and determination of P from auxiliary condition (9).

Table 1 shows results of calculations of the first eigenvalue M_1 of the approximating homogeneous system. It is apparent that the first eigenvalue M_1 depends roughly linearly on the size D of the zone of weakened bonds, i.e., $M_1 \approx D$. The crack reaches the critical state when M_1 reaches the values M. The corresponding critical dimension D_* of the zone of weakened bonds turns out to be roughly equal to the value of M. Using Eq. (8), we can write the following relation in dimensional quantities:

$$d_* = \frac{\pi}{4}\left(\frac{K_{Ic}}{\sigma_u}\right)^2. \tag{10}$$

It should be noted that the critical dimension d_* of the ZWB for a disk-shaped crack is independent of both the applied load p and the size of the initial crack a and is determined only by the material constant m.

The algebraic system which approximates Eq. (6) is solved with a specified value of M and with an increase in the dimension D with a certain step until its critical value D_* is exceeded. The determinant of the system is calculated simultaneously. Here, as might be expected, the determinant was positive as long as D was less than D_*. As soon as D turned out to be greater than D_*, the determinant changed sign and became negative. It should be noted that this fact is convenient to use in solving problems with linear softening, since it makes it possible to solve the problem of crack stability without the use of special methods of determining eigenvalues. One simply observes the sign of the determinant calculated during the solution of the algebraic system which approximates the corresponding integral equation.

Figure 3 shows the stress distribution in a ZWB obtained by solving the approximating system with M_1 = 1 for three ZWB sizes: D = 0.3; 0.6; 0.9. It is apparent that the stress Q(R) increases smoothly from a certain value Q (1) on the edge of the crack (R = 1) to unity at the end of the ZWB (R = B). An increase in the applied load P and a corresponding increase in the size of the ZWB are accompanied by a decrease in the stream Q (1) on the crack edge. This stress takes a minimum value (for each specified value of M) in the critical state. The second line of Table 2 shows values of Q_* (1) in the critical state for different values of M. This data and Eq. (1) make it possible to determine the corresponding dimensionless critical crack opening $\delta_*(1)/\delta_u$, which are shown in the third line of Table 2. Analysis of the data in Table 2 shows that the descending section of the force—displacement curve may be stable for nearly its entire length only in the case of sufficiently small values of M, which corresponds to large crack sizes a or small values of the material constant m. This, in accordance with (8), in turn corresponds to a low fracture toughness for the HAZ metal. In the opposite case, i.e., when the crack size a is small or when the HAZ metal is fairly ductile, the descending section of the force—displacement curve is not completely stable. In the limit of

TABLE 2

M	0,10	0,25	0,33	0,50	1,00	2,00
$Q_*(1)$	0,09	0,12	0,16	0,23	0,39	0,56
$\delta_*(1)/\delta_{II}$	0,91	0,88	0,84	0,77	0,61	0,44

an infinitely small crack, the force—displacement curve generally has no descending section under the stress-controlled loading conditions being examined here, and fracture occurs in accordance with the second assumption made at the beginning of the article. Here, the stress q(r) in the ZWB becomes approximately constant and equal to σ_u.

Figure 4 shows the dependence of the size of the ZWB d/a on the magnitude of the applied load p/σ_u obtained for different values of the parameter M. Despite the fact that local softening of the HAZ metal was assumed near the crack front, the global curve describing the behavior of the metal with a crack has only an ascending section until the critical state is reached. Attainment of this state is indicated in Fig. 4 by the x's. The same pattern is seen in actual tests under stress-controlled loading. The fracture resistance of the HAZ metal with a crack, described by the global curve in Fig. 4, and the corresponding critical loads p_* burn out to be higher, the greater the value of M, i.e., the smaller the initial crack a and the greater the material constant m. The latter is directly proportional to the square of the fracture toughness of the HAZ metal. In the limit of an infinitely small crack, the critical load p_* approaches the ultimate strength σ_u of the defect-free material.

Figure 5 compares normalized critical loads $P_* = p_*/\sigma_u$ for a disk-shaped crack obtained in the present study with normalized critical loads obtained from the most widely used criteria.

The critical load for a disk-shaped crack obtained from the theory of brittle fracture is determined from the Zak formula (see, e.g., [16, 18])

$$p_* = \sqrt{\frac{\pi E\gamma}{2(1-\nu^2)a}}.$$

The corresponding normalized load, with allowance for (2) and (7), is written in the form

$$P_* = 1\sqrt{G}, \tag{11}$$

where $G = a/m$ is the equivalent crack size.

In the case when the stresses in a thin ZWB at the edge of a disk-shaped crack are constant and equal to σ_u — which corresponds to the Leonov—Panasyuk model (see, e.g., [16]) — the size of the zone of weakened bonds d and the crack opening $\delta(a)$ are determined from the following respective formulas, the exact formula depending on the applied load p

$$d = a[1/\sqrt{1-(p/\sigma_u)^2} - 1]; \tag{12}$$

$$\delta(a) = [8(1 \to \nu^2)\sigma_u/\pi E]a[1 - \sqrt{1-(p/\sigma_u)^2}]. \tag{13}$$

Taking a limiting separation of the HAZ boundaries equal to 0.5 σ_u as a constant of the HAZ metal in this model (this value was chosen so that the area under the ideal yield curve with a constant stress σ_u would be equal to the area under the softening curve described by line 2 in Fig. 1b), assuming that the crack opening $\delta(a)$ sometimes (including in the critical state) does not exceed the limiting separation of the HAZ boundaries, i.e., $\delta(a) \leqslant 0.5\ \sigma_u$, and using Eqs. (2), (7), and (13), we obtain the upper estimate for the normalized critical load

$$P_* = \sqrt{4G-1}/(2G), \tag{14}$$

where G can take values greater than 0.5. Insertion of (14) into (12) gives the corresponding upper estimate for the critical size of the ZWB: $d_* = 0.5m(1 - 0.5m/a)$, where $a > 0.5m$. In ccordance with this formula, the lowest upper estimate for d_* occurs in the case of an infinitely large crack and proves to be equal to 0.5m. Use of this value of d_* for 0.5m as the lower estimate of the critical size of the ZWB and its substitution into (12) gives the corresponding lower estimate of the normalized critical load

$$P_* = \sqrt{4G+1}/(2G+1). \tag{15}$$

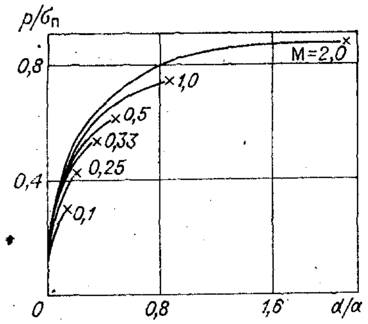

Fig. 4

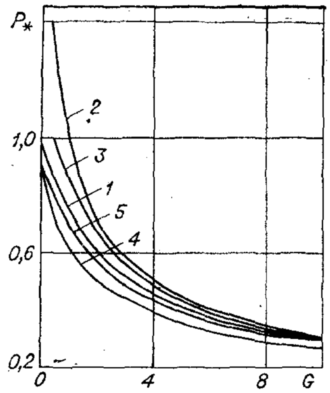

Fig. 5

We can use similar notations to write well-known (see, e.g., [18]) formulas for the critical loads with a disk-shaped crack on the basis of the general integral variational principle of the crack theory proposed in [19]. Here, for the case of a constant crack opening and for the case of proportionality of the crack opening to the applied load, we respectively have the following normalized critical load:

$$P_* = (\sqrt{4G+1}-1)/(2G); \tag{16}$$

$$P_* = 1/\sqrt{G+1}. \tag{17}$$

The loads P_* calculated in this study on the basis of the solution of integral equation (6) with a specified value of M = 1/G exceed the loads determined from Eq. (15) by no more than 1%. Thus, they correspond to curve 1 in Fig. 5. Curves 2-5 correspond to Eqs. (11), (14), (16), and (17). Comparison of curve 1 with curves 2-5 shows that if the HAZ metal is ductile or if the crack radius is small (in other words, if G is small), then the differences in the results will become noticeable. Meanwhile, Eqs. (11) and (14) cease to be valid at values of G respectively less than 1 and 0.5. When G = 0.5, the value of P_* obtained from Eq. (11) is greater than 1. This contradicts the findings of actual tests. The value of P_* obtained from Eq. (14) exceeds the value of P_* obtained in this study by 14%, while the values of P_* obtained from Eqs. (15)-(17) are less than the values obtained in this study by 1, 17, and 7%, respectively. The foregoing comparison shows that for the investigated disk-shaped crack, with linear softening of the material in a thin ZWB, the normalized critical loads can be determined with sufficient accuracy from Eq. (15) without resorting to numerical solution of integral equation (6). Here, the value of the critical load is smaller than the corresponding values obtained from models based on a nondecaying stress-strain curve, while it is larger than the corresponding values obtained from the conservative estimates in [18] — which did not consider the specific character of the behavior of the material at the crack tip.

In conclusion, we should note that actual HAZ metal has a tendency to strain-harden to this or that degree. In connection with this, to correctly describe the behavior of a crack it is generally necessary to use a more complicated idealized curve of force versus displacement of the HAZ boundaries. This more complicated curve includes not only the elastic and linear-softening sections, but also the strain-hardening section. An idealized curve containing the elastic, hardening, and softening sections should obviously lead to an increase in the critical load for the same given size of equivalent crack relative to case examined here, where the curve contained only the elastic and softening sections but had the same area as the curve with three sections. Thus, in the absence of a complete experimental curve plotting force versus displacement of the HAZ boundaries but in the presence of strength characteristic σ_u and fracture toughness characteristic K_{Ic} for the HAZ metal, to obtain the most conservative esimate of the critical load it is necessary to proceed on the basis of the idealized, two-section curve examined in the present study. The limiting irreversible separation of the boundaries of the HAZ for such a curve is evaluated from Eq. (3).

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